Synthesis of Filters for Determination of the Distance Between Two Aircraft

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Abstract: Precise distance measurement between two aircraft is required in many practical cases in radio navigation. It is also important to keep the measured distance's value greater than a critical value in order to ensure flight safety. In this report is realized with the help of two algorithms for precise distance measurement for filter synthesis - a classical and modified one.

Keywords: radionavigation, aircraft (AC), filtration

1. INTRODUCTION

The measurement of the distance between the two objects can be performed by the signals, emitted from the landing system at the approach to the aerodrome (runway), or by the signals, emitted from a satellite navigation system (SNS), when flying and using such systems (Dimitrov, 2014).

In this report are proposed and analyzed the classical and modified algorithm for the synthesis of filters corresponding to the Markov theory of optimal linear filtration in a Gaussian approximation.

2. SYNTHESIS OF FILTERS

On Fig. 1 is shown the geometric position of two aircraft designated with AC_1 and AC_2 in the horizontal plane. At points A, B and C, respectively, are the landing system (LS), AC_1 and AC_2.

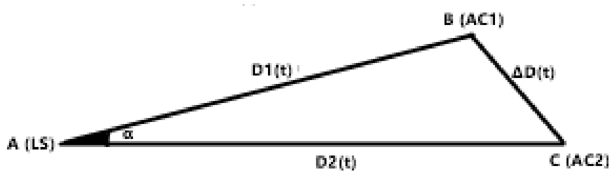

Fig.1. Geometric position of two aircraft and the landing system (LS).

Taking into consideration $\sphericalangle\alpha$ we can notice the distance between points AC_1 and AC_2. Distances respectively are:

- $D_1(t)$ is the distance between LS and AC_1;
- $D_2(t)$ is the distance between LS and AC_2;
- $D(t)$ is the distance between AC_1 and AC_2 which will be evaluated.

In order to ensure the safety of the performed flights, it is assumed that $\Delta D(t)$ can not be less than 50 m.

The two distances $D_1(t)$ and $D_2(t)$ are measured by the signals emitted from the LS. It is assumed that $D_2(t) > D_1(t)$.

Distance information is contained in the signal delays or the changes in the frequency or phase of the signals, depending on the type of landing system which is being used.

The two signals by which $\Delta D(t)$ is determined can be written as:

$$\xi_1(t) = D_1(t) + n_1(t), \tag{1}$$

$$\xi_2(t) = D_2(t) + n_2(t) \tag{2}$$

$$\xi - 1.,t. =, D - 1.$$

$$,t.+, n - 1.(t), \xi - 1.,\mathrm{t.} = \mathrm{D}{-}1.,\mathrm{t.}{+},\mathrm{n}{-}1.(t)$$

Where:

- $D_1(t)$ and $D_2(t)$ are the distances between LS, AC_1 and AC_2.
- $n_1(t)$ and $n_2(t)$ are the two noises that are the errors that measure the distan-ces

$D_1(t)$ and $D_2(t)$ and can be repre-sented as independent White Gaussian Noises (WGN) with zero mathematical expectations $M\{n_1(t)\}=0$, $M\{n_2(t)\}=0$ and delta – shaped correlation functions $k_{ni}(\tau)=\frac{N_i}{2}\delta(\tau)$ where $i=1,2$. Since the distance $\Delta D(t)$ between AC_1 and AC_2 is assumed not to be large, it can be assumed that the spectral densities of the two noises are equal, i.e. $\frac{N_1}{2}=\frac{N_2}{2}=\frac{N}{2}$.

The purpose of the formulated problem is to determine with high accuracy the distance, $\Delta D(t)$ or in other words, to determine the evaluation of the distance $\Delta D(t)$ between AC_1 and AC_2 by the signals (1) and (2).

In solving this problem, except for the assumption that $D_2(t)>D_1(t)$, it is assumed that $\measuredangle\alpha<1°$ and can be solved by the cosine theorem.

If $D_2(t)=26$ km and $D_1(t)=25$ km, then it is obvious that angle alpha is a sharp angle and at $\Delta D(t)=50\text{ m}=0.05$ km from the triangle ΔABC in Fig. 1 it is determined $\cos\alpha$:

$$\cos\alpha=\frac{D_1^2+D_2^2-\Delta D^2}{2D_1D_2}=$$
$$=\frac{25^2+26^2-0,05^2}{2.25.26}\approx 1$$

The angle itself $\measuredangle\alpha<1°$.

At such a small angle, an approximate equality can be written:

$$D_2(t)=D_1(t)+\Delta D(t) \tag{3}$$

It is case (2) can be written as:

$$\xi_2(t)=D_1(t)+\Delta D(t)+n_2(t) \tag{4}$$

In solving the problem, the classical variant of a multidimensional linear filter based on the received signals (1) and (4) (Ivanov,1999); Ivanov, 1986) is used.

The models of the filtered processed $D_1(t)$ and $\Delta D(t)$ are linear and are assumed to be described by a differential equations:

$$\frac{dD_1(t)}{dt}=-\alpha_1 D_1(t)+n_{01}(t) \tag{5}$$

$$\frac{dD(t)}{dt}=-\alpha D(t)+n_0(t) \tag{6}$$

In equation (5), α_1 is the coefficient characterizing the width of the spectrum of fluctuation of the process of $D_1(t)$, and $n_{01}(t)$ is the forming WGN of the process.

The white noise in (5), has mathematical expectation $M\{n_{01}(t)\}=0$ and spectral density $\frac{N_{01}}{2}$.

In (6), α is the coefficient characterizing the width of the spectrum of fluctuation of the process and forming the WGN of the process $\Delta D(t)$.

In vector – matrix form the received signals (observations) have the form:

$$\bar{\xi}=[\xi_1\xi_2]^T=\bar{H}\bar{D}(t)+\bar{n}(t) \tag{7}$$

where: $\bar{D}(t)=[D_1(t)D_2(t)]T$,

$$H=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix},\ a\ \bar{n}(t)=\begin{bmatrix}\frac{N_1}{2} & 0\\ 0 & \frac{N_{01}}{2}\end{bmatrix}.$$

Equations (5) and (6) of the models of the filtered processes are written as:

$$\frac{d\bar{D}(t)}{dt}=\bar{A}(t)\bar{D}(t)+\bar{n}_0(t), \tag{8}$$

where $\bar{D}(t)=[\bar{D}_1(t)\Delta D(t)]T$,

$$A(t)=[-\alpha-\alpha_1]^T \text{ and } \bar{n}(t)=\begin{bmatrix}\frac{N_1}{2} & 0\\ 0 & \frac{N_{01}}{2}\end{bmatrix}.$$

The equation for optimal filtering of the vector $\overline{D}(t)$ described by the vector-matrix equation (8) and the observation (7) for the solved problem, has the form (Ivanov, 1999).

$$\frac{dD(t)}{dt}=\left[R_1(t)\frac{2}{N_1}\left(\xi_1(t)-D_1(t)\right)+\right.$$
$$\left.+R_2(t)\frac{2}{N_2}\left(\xi_2(t)-D_1(t)-\Delta D(t)\right)\right] \tag{9}$$

The synthesized filter according to equation (9) is shown in Fig.2.

The filter is an optimum dual – input filter. At its output, the difference between the estimates $D_2(t)-D_1(t)$ is the estimate $\Delta D(t)$.

A modified variant of a filter is proposed, where the distance $\Delta D(t)$ is determined by observations (1) and (3).

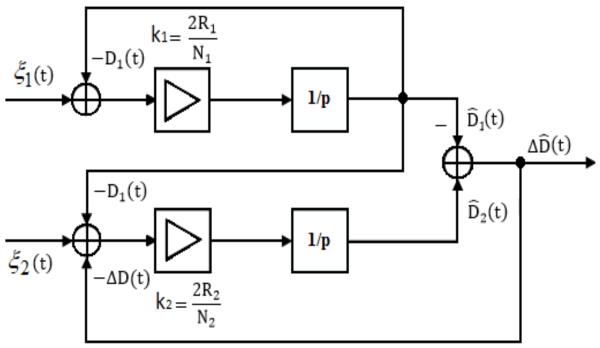

Fig.2 Optimal dual-inlet filter.

The essence of the algorithm by which the filter is synthesized consists of forming a new differential observation (signal) denoted by $\xi(t)$ (Ivanov, 1986); Ivanov, 1987).

$$\xi(t)=\xi_2(t)-\xi_1(t)=\Delta D(t)+n(t) \tag{10}$$

where $n(t)=n_2(t)-n_1(t)$ is WGN with $M\{n(t)\}=0$ and $k_n(\tau)=\frac{N^*}{2}\delta(\tau)$; $\frac{N^*}{2}$ is spectral density of $n(t)$.

A linear model describing the variation of distance $\Delta D(t)$:

$$\frac{dD(t)}{dt}=-\alpha D(t)+n_0(t) \tag{11}$$

where: α is the coefficient, which determines the width of the fluctuation of the spectrum of process $\Delta D(t)$, and $n_0(t)$ is the forming WGN with $M\{n_0(t)\}=0M,n-0(t)=0$ and spectral density $\frac{N_0}{2}$.

In such a model for $\Delta D(t)$ and observation (10), the problem is reduced to quasi – optimal linear filtration, where the quasi – optimality is concluded in the usage of the observation in the differences.

The linear filtration algorithm according to (Ivanov, 1999) has the form:

$$\frac{d\Delta D(t)}{dt}=-\alpha(t)\Delta D(t)+$$
$$+\frac{2R(t)}{N}H(t)\left[\xi(t)-H(t)\Delta D(t)\right] \tag{12}$$

$$\frac{dR(t)}{dt}=2\alpha D(t)-2\alpha R(t)-\frac{2}{N}H^2R^2 \tag{13}$$

where: $R(t)$ is the dispersion of $\Delta D(t)$; $\frac{N}{2}$ is the spectral density of the noise in the observation (10); $H=1$ and $R(0)=\Delta D(t)$.

The filter according to equation (12) has the form shown in Fig. 3 and is a modified single – channel linear filter which uses the formed differential observation $\xi(t)$.

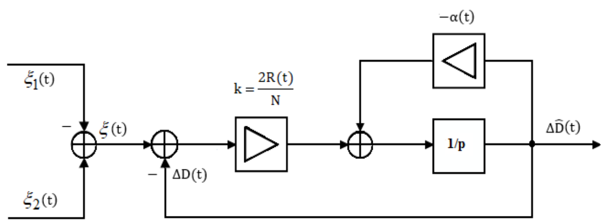

Fig.3 Modified single – channel linear filter.

3. CONCLUSION

The proposed modified filter is quasi – optimal. It is easier for implementation, but the dispersion of the filtration error $R(t)$ at the distance $\Delta D(t)$ should not be expected to be minimal.

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