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Quasi-classical Quantization of Pulsating Strings in Schrödinger spacetime

Hristo Dimov[1,3], Miroslav Radomirov[1], Radoslav Rashkov[1,2] and Tsvetan Vetsov[1,3]

[1]Department of Physics, Sofia University, 5 J. Bourchier Blvd, 1164 Sofia, Bulgaria

[2]Institute for Theoretical Physics, Vienna University of Technology, Wiedner Hauptstr. 8-10, 1040 Vienna, Austria

[3]The Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Moscow region, Russia.

Abstract. In this paper we study a class of classical pulsating string on five dimensional Schrödinger space times five dimensional $T^{1,1}$. This background is obtained by applying TsT transformations to $AdS_5 \times T^{1,1}$ spacetime. After the transformations the new geometry acquires non-trivial antisymmetric B-field, which is the string analogy of a magnetic field. We consider the standard expression for the Nambu-Goto string action and derive the Hamiltonian. Finally, we quantize the theory semi-classically and obtain the wave function, the energy spectrum and the anomalous dimensions of the operators from the dual gauge field theory.

Keywords: string theory, holography, Schrödinger spacetime, pulsating strings, quasi-classical quantisation.

1. INTRODUCTION

In 1998 Juan Maldacena conjectured duality between type II B string theory living in 10 dimensional $AdS_5 \times S^5$ bulk spacetime, and four dimensional conformal gauge theory living on the boundary of that region (Maldacena, 1998; Gubser, 1998; Witten, 1998). More precisely, the correspondence is between the quantum physics of strongly correlated systems to the classical dynamics of gravity in higher dimensions.

The duality is realized by matching between the local fields in the bulk space and the field operators in the quantum theory. Therefore, a scalar field can be dual only to scalar operator, a vector filed A_μ is dual to a current $\hat{J}_\mu$, while the spin-two filed $g_{\mu\nu}$, which is the metric of the bulk theory, is dual to the energy-momentum tensor of the quantum theory (Ramallo, 2015).

Although its impressive achievements the AdS/CFT correspondence in $AdS_5 \times S^5$ can be generalized to include less supersymmetric backgrounds. The latter are more interesting from physical point of view, since they allow us to construct more realistic gauge theories, such as quantum chromodynamics. One way to break the amount of supersymmetry in a given theory is to deform the original AdS space in a certain way, which will also violate the symmetries in the dual CFT. An interesting example is the five dimensional Schrödinger space, which is obtained by applying T-duality – shift – T-duality (TsT) type of transfomations on the AdS_5 space. In this case, the dual CFT is invariant under a non-relativistic Schrödinger group. Moreover, the new Schr/CFT duality can be used to describe strongly correlated non-relativistic systems.

This paper is organized as follows. In Section 2 we obtain the five dimensional Schrödinger space by applying TsT transformations on the coordinates of $AdS_5 \times T^{1,1}$ space. In section 3 we consider the standard Polyakov string action and derive the corresponding equations of motion. Consequently we find simple solutions. In section 4 we start with Nambu-Goto string action from which we find the Hamiltonian of the system. Using perturbation theory we derive the energy spectrum and the anomalous dimension of the dual gauge theory. Finally, in Section 5, we briefly comment on our results.

2. SCHRÖDINGER BACKGROUND

The $Schr_5 \times T^{1,1}$ spacetime is obtained by applying TsT transformations on $AdS_5 \times T^{1,1}$ (Georgiou, 2017; Ouyang, 2017; Ahn, 2018). The metric of the 5-dimensional AdS in global coordinates is given by

$$\frac{ds^2_{AdS}}{l^2} = -\left(1+\frac{\vec{X}^2}{Z^2}\right)dT^2 + \frac{2dTdV + d\vec{X}^2 + dZ^2}{Z^2}. \quad (1)$$

The TsT transformation is defined as follows. First we pick a $U(1)$ isometry direction on the $T^{1,1}$, corresponding to rotations along an angular direction φ, and one isometry direction on the AdS_5, which in this case is T. Next, we perform the following set of transformations (Guica, 2017):

- a T-duality along φ,
- a shift $T \to T + \mu\tilde{\varphi}$, where $\tilde{\varphi}$ is the T-dual coordinate to φ,
- a T-duality back along φ.

Let us apply this set of transformations to the $AdS_5 \times T^{1,1}$ metric $ds^2 = ds^2_{AdS_5} + ds^2_{T^{1,1}}$, where the Kalb-Ramond 2-form field $B_{(2)} = 0$. The metric on $T^{1,1}$ has the form

$$\begin{aligned}\frac{ds^2_{T^{1,1}}}{l^2} = \frac{b}{4}\Bigg[&\sum_{i=1}^{2}\left(d\theta_i^2 + \sin^2\theta_i d\phi_i^2\right) \\ &+ b\left(d\varphi - \sum_{i=1}^{2}\cos\theta_i d\phi_i\right)^2\Bigg],\end{aligned} \quad (2)$$

where $0 \le \varphi < 4\pi,\ 0 \le \theta_i < \pi,\ 0 \le \phi_i < 2\pi$, and $b = 2/3$. After the TsT transformations we derive the $Schr_5$ part of the new metric:

$$\begin{aligned}\frac{ds^2_{Schr_5}}{l^2} = &-\left(1+\frac{\mu^2}{Z^4}+\frac{\vec{X}^2}{Z^2}\right)dT^2 \\ &+ \frac{2dTdV + d\vec{X}^2 + dZ^2}{Z^2}.\end{aligned} \quad (3)$$

while $T^{1,1}$ the part of the metric is left unchanged. Moreover, after the transformation the new background picks up a non-zero antisymmetric B-field:

$$B_{(2)} = \frac{l^2 b\mu}{2\alpha' Z^2} dT \wedge \left(d\varphi - \sum_{i=1}^{2}\cos\theta_i d\phi_i\right). \quad (4)$$

After these preparations, we are ready to derive and analize the string equations of motion in the new background.

3. ANSATZ AND CLASSICAL EQUATIONS OF MOTION

The Plyakov string action in conformal gauge, ($\alpha, \beta = 0,1$ and $M, N = 0,...,9$), is given by

$$\begin{aligned}S = -\frac{1}{4\pi\alpha'}\int d\tau d\sigma\Big(&\sqrt{-h}h^{\alpha\beta}\partial_\alpha X^M \partial_\beta X^N G_{MN} \\ &-\epsilon^{\alpha\beta}\partial_\alpha X^M \partial_\beta X^N B_{MN}\Big),\end{aligned} \quad (5)$$

where $h^{\alpha\beta} = \mathrm{diag}(-1,1)$ and $\epsilon^{01} = -\epsilon^{10} = 1$. In order to obtain pulsating string solutions we consider the following ansatz:

$$\begin{aligned}&T = \kappa\tau,\ \kappa > 0,\ Z = const \ne 0,\ \vec{X} = 0,\ V = 0, \\ &\theta_1 = \theta_1(\tau),\ \ \theta_2 = \theta_2(\tau), \\ &\phi_1 = m_1\sigma,\ \ \phi_2 = m_2\sigma,\ \ \varphi = \phi_3 = m_3\sigma.\end{aligned} \quad (6)$$

In this case, the equations of motion for V, X, ϕ_1, ϕ_2 and φ, are trivially satisfied, while the equations for T, Z, θ_1, θ_2 stay relevant. Let us begin with the equation along T

$$m_1 \sin\theta_1(\tau)\dot{\theta}_1(\tau) + m_2 \sin\theta_2(\tau)\dot{\theta}_2(\tau) = 0. \quad (7)$$

It can immediately be written in the following useful form

$$m_1 \cos\theta_1(\tau) + m_2 \cos\theta_2(\tau) = A, \qquad m_{1,2} \ne 0.$$

If we choose $\cos\theta_1$ and $\cos\theta_2$ to be symmetric with respect to 0, then one requires $-q_1 \le \cos\theta_1 \le q_1,\ \ (0 \le q_1 \le 1)$ and

$-q_2 \le \cos\theta_2 \le q_2,\ \ (0 \le q_2 \le 1)$.

The latter restrictions imply $A = 0$ and $|m_1|q_1 = |m_2|q_2$. Therefore, equation (7) takes a certain form, which we call the pulsating strung condition

$$m_1\cos\theta_1(\tau)+m_2\cos\theta_2(\tau)=0. \qquad (8)$$

The EoM along Z is given by

$$Z^2=\frac{2\alpha'\kappa\mu}{bm_3}=const, \quad m_3>0, \qquad (9)$$

while the EoMs along $\theta_i,\ i=1,2$ yield

$$\ddot{\theta}_i(\tau)+ m_i\sin\theta_i(\tau)\left(m_i\cos\theta_i(\tau)+bm_3-\frac{2\kappa\mu}{\alpha' Z^2}\right)=0. \qquad (10)$$

We should also supplement the equations of motion together with the Virasoro constraints:

$$G_{MN}\left(\dot{X}^M\dot{X}^N+X'^M X'^N\right)=0, \quad G_{MN}\dot{X}^M X'^N=0, \qquad (11)$$

where the first equation explicitly reads

$$\dot{\theta}_1^2(\tau)+\dot{\theta}_2^2(\tau)+m_1^2\sin^2\theta_1(\tau)+m_2^2\sin^2\theta_2(\tau) +bm_3^2-\frac{4|G_{00}|\kappa^2}{b}=0, \qquad (12)$$

while the second Virasoro constraint is trivially satisfied.

Simple solutions can be obtained if one considers the following special case $m_1=m_2=m$. Therefore, Eqs. (8), (10), (12) becomes

$$\cos\theta_1+\cos\theta_1=0 \Rightarrow \theta_2=\pi-\theta_1, \qquad (13)$$

$$\ddot{\theta}_i+m\sin\theta_i\left(m\cos\theta_i+bm_3-\frac{2\kappa\mu}{\alpha' Z^2}\right)=0, \qquad (14)$$

$$\dot{\theta}_1^2+\dot{\theta}_2^2+m^2\left(\sin^2\theta_1+\sin^2\theta_2\right) +bm_3^2-\frac{4|G_{00}|\kappa^2}{b}=0, \qquad (15)$$

Using $\theta_2=\pi-\theta_1$ Eq. (14) for θ_1 and θ_2 read

$$\ddot{\theta}_1+m\sin\theta_1\left(m\cos\theta_1+bm_3-\frac{2\kappa\mu}{\alpha' Z^2}\right)=0, \qquad (16)$$

$$-\ddot{\theta}_1+m\sin\theta_1\left(-m\cos\theta_1+bm_3-\frac{2\kappa\mu}{\alpha' Z^2}\right)=0, \qquad (17)$$

and Eq. (15) changes to

$$\dot{\theta}_1^2+m^2\sin^2\theta_1=\frac{2|G_{00}|\kappa^2-b^2m_3^2}{2b}. \qquad (18)$$

Now we can subtract (16) and (17) to find

$$\ddot{\theta}_1+m^2\sin\theta_1\cos\theta_1=0. \qquad (19)$$

Multiplying by $\dot{\theta}_1$, one has

$$\dot{\theta}_1\ddot{\theta}_1+m^2\dot{\theta}_1\sin\theta_1\cos\theta_1=0,$$

or

$$\frac{d}{d\tau}\left(\dot{\theta}_1^2+m^2\sin^2\theta_1\right)=0, \Rightarrow \quad \dot{\theta}_1^2+m^2\sin^2\theta_1=N^2. \qquad (20)$$

The constant N follows from the Virasoro equation (18)

$$N^2=\frac{2|G_{00}|\kappa^2-b^2m_3^2}{2b}, \quad 2|G_{00}|\kappa^2>b^2m_3^2. \qquad (21)$$

Now we are in a position to integrate (20), namely

$$\int_0^{\theta_1}\frac{d\theta_1}{\sqrt{1-\frac{m^2}{N^2}\sin^2\theta_1}}=F\left(\theta_1,\frac{m}{N}\right)=\pm|N|\int_0^{\tau}d\tau. \qquad (22)$$

Finally, the solution is given in terms of the Jacobi elliptic sine:

$$\sin\theta_1(\tau)=\pm\mathrm{sn}\left(|N|\tau,\frac{m}{N}\right), \qquad (23)$$

or explicitly for θ_1 and θ_2:

$$\theta_1(\tau)=\pm\arcsin\left(\mathrm{sn}\left(|N|\tau,\frac{m}{N}\right)\right), \quad \theta_2(\tau)=\pi-\theta_1(\tau). \qquad (24)$$

One notes that the obtain solution are periodic functions.

3. ENERGY CORRETIONS AND ANOMALOUS DIMENSIONS

In order to find the energy of the pulsating string configuration and its higher order corrections we consider the bosonic part of the Nambu-Goto string action

$$S_{NG}=-\frac{1}{2\pi\alpha'}\int d\tau d\sigma\sqrt{-\det\left(\gamma_{\alpha\beta}-b_{\alpha\beta}\right)}, \qquad (25)$$

where $\gamma_{\alpha\beta}$ and $b_{\alpha\beta}$ are the induced metric, and B-field given by

$$\begin{aligned}\gamma_{\alpha\beta} &= G_{MN}\partial_\alpha X^M \partial_\beta X^N,\\ b_{\alpha\beta} &= B_{MN}\partial_\alpha X^M \partial_\beta X^N.\end{aligned} \tag{26}$$

Now, the first step towards finding the spectrum is to make a pullback of the line element of the metric of $Schr_5 \times T^{1,1}$ to the subspace, where string dynamics takes place. The result for the metric is

$$\begin{aligned}ds^2 = l^2 \Bigg(&-|G_{00}|\,dT^2 + \sum_{i,j=1}^{2} G_{ij} d\theta^i d\theta^j \\ &+ \sum_{k,h=1}^{3} \hat{G}_{kh} d\phi^k d\phi^h \Bigg),\end{aligned} \tag{27}$$

where we have defined the following quantities

$$|G_{00}| = 1 + \frac{\mu^2}{Z^4}, \quad (G_{ij}) = \frac{b}{4}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{28}$$

and

$$\left(\hat{G}_{kh}\right) = \frac{b}{4}\begin{pmatrix} b\cos^2\theta_1 + \sin^2\theta_1 & b\cos\theta_1\cos\theta_2 & -b\cos\theta_1 \\ b\cos\theta_1\cos\theta_2 & b\cos^2\theta_2 + \sin^2\theta_2 & -b\cos\theta_2 \\ -b\cos\theta_1 & -b\cos\theta_2 & b \end{pmatrix}.$$

Taking into count the ansatz (6) one can also find the components of the induced metric and the B-field on the worldsheet:

$$\frac{ds_{ws}^2}{l^2} = \left(-|G_{00}|\kappa^2 + \sum_{i,j=1}^{2} G_{ij}\dot{\theta}^i\dot{\theta}^j \right) d\tau^2 + |\vec{m}|^2\, d\sigma^2, \tag{29}$$

$$B_{(2)} = 2l^2 B_{\tau\sigma} d\tau \wedge d\sigma, \quad B_{\tau\sigma} = \frac{1}{2}\sum_{i=1}^{3} b_{0i}\kappa m_i, \tag{30}$$

where

$$\begin{aligned}|\vec{m}|^2 &= \sum_{k,h=1}^{3} \hat{G}_{kh} m_k m_h = \frac{b}{4}\left(\sum_{j=1}^{2} m_j^2 \sin^2\theta_j + b m_3^2 \right) \\ &= \frac{b}{4}\left(m_1^2 + m_2^2 + b m_3^2 - 2 m_i^2 \cos^2\theta_i \right) \\ &= \frac{b}{4} P_2\left(\cos\theta_i\right) > 0,\end{aligned} \tag{31}$$

$$\begin{aligned}&b_{01} = -\frac{b\mu}{2\alpha' Z^2}\cos\theta_1, \quad b_{02} = -\frac{b\mu}{2\alpha' Z^2}\cos\theta_2, \\ &b_{03} = \frac{b\mu}{2\alpha' Z^2}.\end{aligned} \tag{32}$$

After simplifying the sum in the previous equation and taking into account Eq. (8) one finds

$$B^2 = B_{\tau\sigma}^2 = \frac{b^2\mu^2\kappa^2 m_3^2}{16\alpha'^2 Z^4}. \tag{33}$$

Now we can write the Nambu-Goto action (25) in terms of the notations we introduced above:

$$S_{NG} = -\frac{l^2}{\alpha'}\int d\tau \sqrt{|\vec{m}|^2 \left(|G_{00}|\kappa^2 - \sum_{i,j=1}^{2} G_{ij}\dot{\theta}^i\dot{\theta}^j \right) - B^2}, \tag{34}$$

where $l^2/\alpha' = \sqrt{\lambda}$ is the 't Hooft coupling constant. The next step towards the spectrum is to consider the Hamiltonian formulation of the problem. In our case, the canonical momenta are given by

$$\Pi_k = \frac{\partial L}{\partial \dot{\theta}^k} = \frac{\sqrt{\lambda}\,|\vec{m}|^2 \sum_{i=1}^{2} G_{ki}\dot{\theta}^i}{\sqrt{|\vec{m}|^2 \left(|G_{00}|\kappa^2 - \sum_{i,j=1}^{2} G_{ki}\dot{\theta}^i\dot{\theta}^j \right) - B^2}}. \tag{35}$$

Using the Legendre transformed Lagrangian, $L = \Pi_k \dot{\theta}^k - H$, we find the square of the Hamiltonian:

$$\begin{aligned}H^2 = &\frac{|\vec{m}|^2 |G_{00}|\kappa^2 - B^2}{|\vec{m}|^2} \sum_{i,j=1}^{2} G^{ij}\Pi_i\Pi_j \\ &+ \lambda\left(|\vec{m}|^2 |G_{00}|\kappa^2 - B^2 \right).\end{aligned} \tag{36}$$

As in other cases of pulsating strings in holography, we observe that H^2 looks like a point-particle Hamiltonian, in which the last term serves as a potential

$$U\left(\theta_1, \theta_2\right) = |\vec{m}|^2 |G_{00}|\kappa^2 - B^2. \tag{37}$$

The procedure we will follow involves a perturbative expansion in the small coupling constant λ. The semiclassical quantization of the pulsating string then acquires corrections to the energy. According to the AdS/CFT duality, the anomalous dimension of the corresponding SYM operators are directly related to the corrections of the energy (Dimov, 2019).

The kinetic term of the Hamiltonian (36) is considered as a two dimensional Laplace-Beltrami operator

$$\sum_{i,j=1}^{2} G^{ij}\Pi_i\Pi_j \to \Delta = \frac{1}{\sqrt{|G|}}\partial_i\left(\sqrt{|G|}G^{ij}\partial_j\right) = \frac{4}{b}\left(\frac{\partial^2}{\partial\theta_1^2}+\frac{\partial^2}{\partial\theta_2^2}\right), \quad (38)$$

where $|G| = \det\left(G_{ij}\right)$. Collecting the results and using the above notations the Schrödinger equation for the wave function $\psi(\theta_1,\theta_2)$ becomes

$$\frac{4\left(|G_{00}|\kappa^2|\vec{m}|^2 - B^2\right)}{b|\vec{m}|^2}\left(\frac{\partial^2}{\partial\theta_1^2}+\frac{\partial^2}{\partial\theta_2^2}\right)\psi = -E^2\psi, \quad (39)$$

where the squared of the energy is

$$E^2 = \frac{4}{b}\left(E_1^2 + E_2^2\right). \quad (40)$$

Using standard separation of the variables $\psi(\theta_1,\theta_2) = \psi_1(\theta_1)\psi_2(\theta_2)$, one easily obtains two differential equations

$$\frac{|G_{00}|\kappa^2|\vec{m}|^2 - B^2}{|\vec{m}|^2}\frac{d^2}{d\theta_i^2}\psi_i(\theta_i) = -E_i^2\psi_i(\theta_i). \quad (41)$$

Introducing a new variables $-1 \le z_i = \cos\theta_i \le 1$ and taking into account (31) and

$$\frac{d}{d\theta_i} = -\sqrt{1-z_i^2}\,\frac{d}{dz_i},\quad \frac{d^2}{d\theta_i^2} = \left(1-z_i^2\right)\frac{d^2}{dz_i^2} - z_i\frac{d}{dz_i},$$

one can rewrite Eq. (41) as

$$\left(\left(1-z_i^2\right)\frac{d^2}{dz_i^2} - z_i\frac{d}{dz_i} + \frac{E_i^2}{|G_{00}|\kappa^2}\frac{P_2\left(z_i\right)}{P_2\left(z_i\right)-\beta^2}\right) \times\psi_i\left(z_i\right) = 0. \quad (42)$$

where $\beta^2 = 4B^2/b|G_{00}|\kappa^2$. We investigate the following case:

Let the polynomial $P_2\left(z_i\right)-\beta^2$ has two different real roots z_i^* and $-z_i^*$, where

$$z_i^* = \sqrt{\frac{m_1^2+m_2^2+bm_3^2-\beta^2}{2m_i^2}}, \quad (43)$$

such that $z_i^* > 1$ and thereby $|\cos\theta_i| = |z_i| < z_i^*$. Hence, one can expand the following expression such as

$$\frac{P_2\left(z_i\right)}{P_2\left(z_i\right)-\beta^2} = 1 + \frac{\beta^2}{2m_i^2 z_i^{*2}}\left(1+\frac{z_i^2}{z_i^{*2}}+\frac{z_i^4}{z_i^{*4}}+\ldots\right). \quad (44)$$

Let us consider the approximation

$$\frac{P_2\left(z_i\right)}{P_2\left(z_i\right)-\beta^2} \approx 1 + \frac{\beta^2}{2m_i^2 z_i^{*2}}. \quad (45)$$

It reduces Eq. (42) to the Chebyshev's equation with solution given by the Chebyshev polynomials of the first kind $T_n\left(z\right)$, namely

$$\left(\left(1-z_i^2\right)\frac{d^2}{dz_i^2} - z_i\frac{d}{dz_i} + n_i^2\right)C_i T_{n_i}\left(z_i\right) = 0, \quad (46)$$

With

$$\psi_i\left(\theta_i\right) = C_i T_{n_i}\left(\cos\theta_i\right) = C_i\cos\left(n_i\theta_i\right),\quad n_i = \mathbb{N}. \quad (47)$$

The energy spectrum is determined by the condition

$$E_i^2 = \left(1 - \frac{\beta^2}{m_1^2+m_2^2+bm_3^2}\right)|G_{00}|\kappa^2 n_i^2, \quad (48)$$

where we used Eq. (43). Furthermore, the wave function

$$\psi\left(\theta_1,\theta_2\right) = \psi_1\left(\theta_1\right)\psi_2\left(\theta_2\right) = C\cos\left(n_1\theta_1\right)\cos\left(n_2\theta_2\right) \quad (49)$$

has to be normalized, i.e.

$$1=\frac{b}{4}\int_0^\pi\int_0^\pi\left|\psi\left(\theta_1,\theta_2\right)\right|^2 d\theta_1 d\theta_2$$
$$=\frac{bC^2}{4}\int_0^\pi\int_0^\pi\cos^2\left(n_1\theta_1\right)\cos^2\left(n_2\theta_2\right)d\theta_1 d\theta_2=\frac{bC^2\pi^2}{16}. \quad (50)$$

The full energy (40) becomes

$$E^2=\left(1-\frac{\beta^2}{m_1^2+m_2^2+bm_3^2}\right)\frac{4\left|G_{00}\right|\kappa^2}{b}\left(n_1^2+n_2^2\right). \quad (51)$$

The first order energy correction can be calculated from the potential

$$\delta E^2=\frac{\lambda b}{4}\int_0^\pi\int_0^\pi\left|\psi\left(\theta_1,\theta_2\right)\right|^2 U\left(\theta_1,\theta_2\right)d\theta_1 d\theta_2$$
$$=\lambda\frac{b\left|G_{00}\right|\kappa^2}{8}\left(m_1^2+m_2^2+2bm_3^2-2\beta^2\right). \quad (52)$$

Combining Eqs. (51) and (52) we find the total energy

$$E_{tot}=\kappa\left(\frac{4\left|G_{00}\right|}{b}\left(1-\frac{\beta^2}{m_1^2+m_2^2+m_3^2+}\right)\left(n_1^2+n_2^2\right)\right.$$
$$\left.+\lambda\frac{b\left|G_{00}\right|}{8}\left(m_1^2+m_2^2+2bm_3^2-2\beta^2\right)\right)^{\frac{1}{2}}. \quad (53)$$

It can be expanded up to first order in λ

$$E_{tot}=E+\Delta+O\left(\lambda^2\right). \quad (54)$$

From the last expression one immediately finds the anomalous dimension Δ of the operators from the dual gauge theory:

$$\Delta=\frac{b\left|G_{00}\right|\kappa\left(m_1^2+m_2^2+2bm_3^2-2\beta^2\right)}{32}$$
$$\times\left(\frac{\left|G_{00}\right|}{b}\left(1-\frac{\beta^2}{m_1^2+m_2^2+bm_3^2}\right)\left(n_1^2+n_2^2\right)\right)^{-\frac{1}{2}}\lambda. \quad (55)$$

One can also consider further terms in the approximation (44), which leads to similar analyzes of the energy spectrum, but in this case the wave function is given by Mathieu sine and cosine special function.

CONCLUSION

Our investigation is inspired by the magnificent duality between gravitational (string) theories and conformal gauge theories. Specifically, we focus on non-relativistic gravitational backgrounds, namely Schrödinger spacetimes, which can be used to describe strongly correlated non-relativistic quantum systems. The key point of such non-relativistic backgrounds is the fact that they reduce the supersymmetry and the conformal symmetry of the initial system considerably. This allows to construct more realistic gauge models.

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