



An Approach for Engineering Tuning of a PID-controller with a Dynamic Second Order Object

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Abstract. An approach is proposed for engineering set up of the PID-controller with a dynamic second order object. A problem solution is offered by solving the characteristic equation. As a result of the third row dynamic system analysis, the set up parameters of the PID-controller are calculated. The transitional processes of the closed system (object-controller) are dealt with by tuning and disturbance. For the transitional process by tuning, overshoot $\sigma = 4,0 \%$ occurs. If a comparison of the overshoot of the transitional process by assignment (obtained in simulation) is made, 0,6% inaccuracy is observed in theory. Therefore, the proposed approach for engineering set up for a PID-controller with a second-order dynamic object is suitable for application in third-order dynamic systems analysis.

Keywords: PID-controller, Dynamic second order object.

1. INTRODUCTION

Third order dynamic systems can be obtained in the following cases:

- object two aperiodic links, operating with I-controller;
- object two aperiodic links, working with PI-controller;
- object two aperiodic links, working with PID-controller with perfect differentiation;
- object three aperiodic links, operating with a P-controller;
- object three aperiodic links, working with a PD-controller with perfect differentiation;
- object oscillating link, operating with PI-controller;
- object oscillating link, operating with PID-controller with ideal differentiation and
- object oscillating link, operating with a real-time differentiation first-order PD-controller.

2. SET UP PROBLEMS OF CONTROLLERS IN THIRD ORDER SYSTEMS

Third order dynamic systems are often used in industrial automation systems for a variety of production processes, but due to

their complexity, few authors have attempted to do theoretical research on them. The complexity is that the roots of the characteristic equation of the closed ACS (automatic control system) is three, and it is not clear how the third real root influences the stability of the system, and hence the indicators of quality of the transitional processes.

3. POSSIBLE OPTIONS FOR SOLUTION OF THE ASSIGNED TASK

In analyzing third order dynamic systems, the determination of dependencies between quality indicators and system parameters is considerably more complicated. One of the possible options for solving the task is through the use of Prof. Vishnegradski's diagram (Naplatanov, 1976). The diagram he suggests allows estimating not only sustainability but also some key quality indicators. In the study of dynamic systems of third order, he concluded that the nature of the transitional process can be determined without solving the characteristic equation of

the system. For this purpose, it is sufficient for hyperbola built according to its parameters - X and Y to be supplemented with three auxiliary curves (Naplatanov, 1976). He has given an original word formulation of his criterion, which states: To be a dynamic third-order system sustainable, it is both necessary and sufficient to fulfill the following two conditions: 1. All the coefficients of the characteristic equation must be positive; 2. The average output minus the output of the final coefficients of the characteristic equation of the system must be positive. Failure to comply with these conditions will make the third order dynamic system unstable or at the limit of resistance.

Other possible options for solving this task are by using Ziegler & Nicols first method, Koppelovich's nomograms and nomograms (Hinov et al., 1978). These are methods for determining the parameters to set up the controllers by known data for the transitional characteristic of the control object (Dragotinov et al., 2008; Terziyski et al., 2015).

The purpose of this article is to offer an engineering set up for a proportional-integral-differential PID-controller with a dynamic second order object.

4. PROBLEM SOLUTION BY SOLVING THE CHARACTERISTIC EQUATION

Fig. 1 shows the structural diagram of a ACS comprising a second order object (two aperiodic units) and a PID-controller.

The transmission function of the closed system (Fig. 1) regarding the assignment is the type

The transmission function of the closed system (Fig. 1) regarding the disturbance is the type

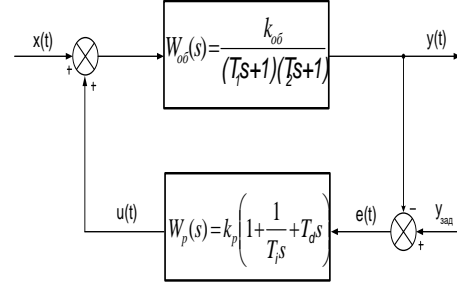


Fig. 1 A system with a second order object and a PID-controller

$$\begin{aligned}
 W_{\text{aad}}(s) &= \frac{Y(s)}{Y_{\text{aad}}(s)} = \frac{W_{\text{oo}}(s) \cdot W_p(s)}{1 + W_{\text{oo}}(s) \cdot W_p(s)} = \\
 &= \frac{\frac{k_{\text{oo}}}{(T_1s+1)(T_2s+1)} \cdot k_p \left(\frac{T_i T_d s^2 + T_i s + 1}{T_i s} \right)}{1 + \frac{k_{\text{oo}}}{(T_1s+1)(T_2s+1)} \cdot k_p \left(\frac{T_i T_d s^2 + T_i s + 1}{T_i s} \right)} = \\
 &= \frac{T_i T_d s^2 + T_i s + 1}{\frac{T_1 T_2 T_i}{k_{\text{oo}} k_p} s^3 + \frac{(T_1 + T_2 + k_{\text{oo}} k_p T_d) T_i}{k_{\text{oo}} k_p} s^2 + \frac{(1 + k_{\text{oo}} k_p) T_i}{k_{\text{oo}} k_p} s + 1}. \quad (1)
 \end{aligned}$$

$$\begin{aligned}
 W_x(s) &= \frac{Y(s)}{X(s)} = \frac{W_{\text{oo}}(s)}{1 + W_{\text{oo}}(s) \cdot W_p(s)} = \\
 &= \frac{\frac{k_{\text{oo}}}{(T_1s+1)(T_2s+1)}}{1 + \frac{k_{\text{oo}}}{(T_1s+1)(T_2s+1)} \cdot k_p \left(\frac{T_i T_d s^2 + T_i s + 1}{T_i s} \right)} = \\
 &= \frac{T_i}{k_p} \cdot \frac{s}{\frac{T_1 T_2 T_i}{k_{\text{oo}} k_p} s^3 + \frac{(T_1 + T_2 + k_{\text{oo}} k_p T_d) T_i}{k_{\text{oo}} k_p} s^2 + \frac{(1 + k_{\text{oo}} k_p) T_i}{k_{\text{oo}} k_p} s + 1}. \quad (2)
 \end{aligned}$$

We propose that the analysis of the third-order dynamic system be carried out with a successively connected oscillating and aperiodic link, i.

$$W(s) = \frac{1}{T_o^2 s^2 + 2\xi T_o s + 1} \cdot \frac{1}{Ts + 1}. \quad (3)$$

Assuming that the time constant of the aperiodic link (first order low pass filter) is

equal to the time constant of the oscillating link, i. $T = T_0$ is obtained

$$W(s) = \frac{1}{T_0^2 s^2 + 2\xi T_0 s + 1} \cdot \frac{1}{T_0 s + 1}. \quad (4)$$

For the polynomial in the denominator of expression (4) the characteristic equation is obtained

$$(T_0^2 s^2 + 2\xi T_0 s + 1)(T_0 s + 1) = T_0^3 s^3 + (2\xi + 1)T_0^2 s^2 + (2\xi + 1)T_0 s + 1. \quad (5)$$

If we equal the corresponding coefficients in front of s^3 , s^2 etc. from the characteristic equation (5) to the coefficients of s^3 , s^2 etc. of the polynomial in the denominator of expression (1), the transfer function of the closed system regarding the assignment will have the final appearance

$$W_{3ad}(s) = k_{3ad} \cdot \frac{T_i T_d s^2 + T_i s + 1}{T_0^3 s^3 + (2\xi + 1)T_0^2 s^2 + (2\xi + 1)T_0 s + 1}, \quad (6)$$

where $k_{3ad} = 1$ is called a coefficient of the system assignment.

The transmission function of the closed disturbance system will have the final appearance

$$W_x(s) = k_x \cdot \frac{T_0 s}{T_0^3 s^3 + (2\xi + 1)T_0^2 s^2 + (2\xi + 1)T_0 s + 1}, \quad (7)$$

where

$$k_x = \frac{T_i}{k_p} \cdot \frac{1}{T_0} = \frac{T_i}{k_p} \cdot \sqrt[3]{\frac{k_{o\sigma} k_p}{T_1 T_2 T_i}} = \sqrt[3]{\frac{T_i^2 k_{o\sigma}}{k_p^2 T_1 T_2}} \text{ is}$$

called the system disturbance factor.

By comparing the coefficients in front of the corresponding degrees of s in the polynomials of expressions (1) and (6), dependencies between the parameters of the transition process and the parameters of the system can be determined. Equivalent time constant is

$$T_o = \sqrt[3]{\frac{T_1 T_2 T_i}{k_{o\sigma} k_p}}. \quad (8)$$

Similarly, the attenuation coefficient ξ is determined. For it two expressions of s^2 and s of (6) are obtained.

The first expression that can be determined ξ is

$$(2\xi + 1)T_0^2 = \frac{(T_1 + T_2 + k_{o\sigma} k_p T_d) T_i}{k_{o\sigma} k_p}. \quad (9)$$

If we only express ξ we obtained

$$\xi = \frac{1}{2} \left[\frac{(T_1 + T_2 + k_{o\sigma} k_p T_d) T_i}{T_0^2 k_{o\sigma} k_p} - 1 \right]. \quad (10)$$

The second expression from which can be determined ξ is

$$(2\xi + 1)T_0 = \frac{(1 + k_{o\sigma} k_p) T_i}{k_{o\sigma} k_p}. \quad (11)$$

If we express only ξ it is obtained

$$\xi = \frac{1}{2} \left[\frac{(1 + k_{o\sigma} k_p) T_i}{T_0 k_{o\sigma} k_p} - 1 \right]. \quad (12)$$

If the expressions (9) and (11) are divided into one another, it is obtained

$$T_o = \frac{T_1 + T_2 + k_{o\sigma} k_p T_d}{1 + k_{o\sigma} k_p}. \quad (13)$$

If the expressions (10) and (12) are equal, then

$$\frac{1}{2} \left[\frac{(T_1 + T_2 + k_{o\sigma} k_p T_d) T_i}{T_0^2 k_{o\sigma} k_p} - 1 \right] = \frac{1}{2} \left[\frac{(1 + k_{o\sigma} k_p) T_i}{T_0 k_{o\sigma} k_p} - 1 \right] \quad (14)$$

and then simplified, an expression of the type (13) is obtained. This confirms that the expressions (8) and (13) are equal.

$$T_o = \sqrt[3]{\frac{T_1 T_2 T_i}{k_{o\sigma} k_p}} = \frac{T_1 + T_2 + k_{o\sigma} k_p T_d}{1 + k_{o\sigma} k_p}. \quad (15)$$

If the expression (15) is solved regarding the time constant of integration T_i , the following formula is obtained

$$T_i = \frac{(T_1 + T_2 + k_{o\sigma} k_p T_d)^3 k_{o\sigma} k_p}{T_1 T_2 (1 + k_{o\sigma} k_p)^3}. \quad (16)$$

The time constant of differentiation T_d can be determined, following formula (10).

$$T_d = \frac{(2\xi + 1)T_0^2 k_{o\sigma} k_p - (T_1 + T_2)T_i}{k_{o\sigma} k_p T_i} \quad (17)$$

The proportionality coefficient of the controller k_p can be determined by expression (11), ie.

$$k_p = \frac{T_i}{[(2\xi + 1)T_0 - T_i]k_{o\sigma}} \quad (18)$$

Example: Transitional process of object is given with two aperiodic links. The following algorithm performs the following:

1. Take the transitional process of the object that is smooth and normalizing.

2. Since the object model is of second order (two consecutively connected aperiodic links with equal time constants) - the transitional characteristic is monotone with a transient delay, it is chosen to approximate the method of Ormans (Badev, 2013). After the approximation, it is determined: $k_{o\sigma} = 1$, $T_1 = T_2 = 19.5$ sec.

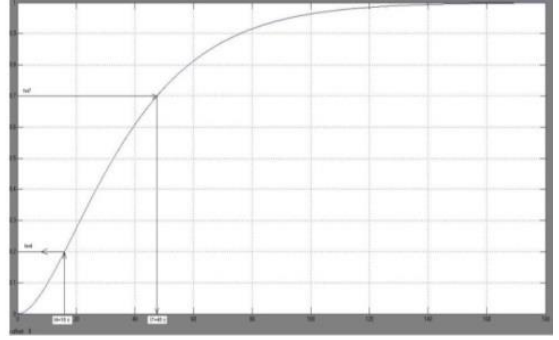


Fig. 2 Transitional process of control object

3. By the expressions (16), (17) and (18), the set up parameters of the PID-controller are calculated using an iteration procedure.

First, calculate the value of k_p , at a set value of $T_i \approx 3\tau_a$, where τ_a is determined after a Cupfmuler approximation

$$k_p = \frac{T_i}{[(2\xi + 1)T_0 - T_i]k_{o\sigma}} = 1.027.$$

Then calculate the value of T_d again at the same set value of T_i

$$T_d = \frac{(2\xi + 1)T_0^2 k_{o\sigma} k_p - (T_1 + T_2)T_i}{k_{o\sigma} k_p T_i} = 4.24 \text{ sec}$$

Finally T_i is calculated, if its value is close to the one above, the calculation procedure is terminated.

$$T_i = \frac{(T_1 + T_2 + k_{o\sigma} k_p T_d)^3 k_{o\sigma} k_p}{T_1 T_2 (1 + k_{o\sigma} k_p)^3} = 25.793 \text{ sec}$$

If the value of T_i differs greatly from the set above, the calculation procedure starts from the beginning by selecting a value T_i , of so to minimize the difference between the set value and the value obtained.

4. By formula (12) the damping factor ξ is calculated and approximately what is the value of the overshoot σ from [3]

$$\xi = \frac{1}{2} \left[\frac{(1 + k_{os} k_p) T_i}{T_0 k_{os} k_p} - 1 \right] = 0,7$$

$$\sigma^2 = \exp \left(- \frac{\xi}{\sqrt{1 - \xi^2}} \cdot 2\pi \right) = 0,0021$$

or only $\sigma = 4,6 \%$.

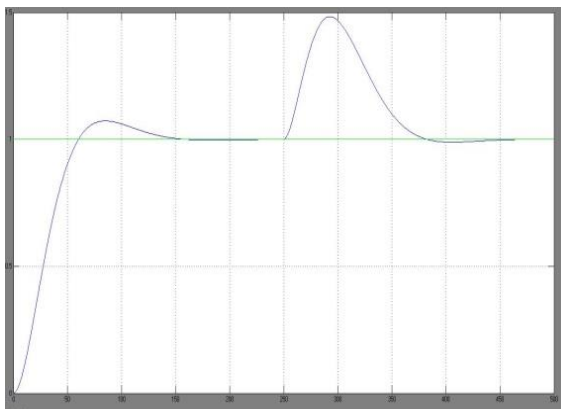


Fig. 3 Transitional processes by assignment and by disturbance

5. Determine the maximum dynamic deviation y_1 in the expression given in [3]

$$y_1 = \exp \left(- \frac{\xi}{\sqrt{1 - \xi^2}} \right) = 0,37.$$

6. If any of the above two parameters does not meet the prerequisites for quality, set up the controller.

The transitional processes of the closed system (Fig. 1) by assignment and by disturbance are shown in Fig. 3. For the transitional process by assignment, overshoot $\sigma = 4,0 \%$ occurs. If a comparison of the overshoot of the transitional process by

assignment (obtained in simulation) is made, 0,6 % inaccuracy is observed in theory. Therefore, the proposed sub-process for engineering set up of a PID-controller with a dynamic object with two aperiodic links is suitable for use in the analysis of third-order dynamic systems.

CONCLUSIONS

An approach is proposed for engineering set up of the PID-controller with a dynamic second order object (two aperiodic links with equal time constants). There is a proposal to solve the problem by solving the characteristic equation. As a result of the analysis of the third order dynamic system, the set up parameters of the PID-controller are calculated.

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