Effective Methods for Calculating Modes of Multilayer Waveguides

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Abstract. In this paper we propose a general electrodynamic model for calculating the modes of multi-layer anisotropic 2D and 3D waveguides. In the numerical simulations for solving the system of private differential equations, the three most frequently used numerical approaches were applied and compared at the time of calculation: FD - Finite difference method, Galerkin's method with a specially proposed system of pseudo-orthogonal functions and TMM - method of transformation matrices.

Keywords: multilayer waveguides, FD method, FE method, TMM method.

1. INTRODUCTION

Multilayer fibers can transmit multiple times more information than one layer waveguides and therefore are goal of many studies. Multilayer waveguides are made up of several layers, each layer having a different refractive index and dielectric permeability. Several different signals can be transmitted over a long distance with minimal loss.

In modelling, we assume that the radiation is presented as an electromagnetic (EM) wave, and the mediums of the layers in the form of a physical carrier characterized by a certain set of optical parameters. This consideration is valid in more than 98% of the practical application of waveguides. Maxwell's equations describe the wave propagation well in these cases.

In modelling 2D multilayer waveguides (the EM wave propagates along the Oz axis), when the media of each "j-th" layer is presented by diagonal tensors of permittivity $[\varepsilon]$ and $[\mu]$ permeability, the following equations for transverse electrical (TE) and transverse magnetic (TM) modes are used:

For TE mods $\vec{E} = (0, E_{yj}(x,z), 0)$:

$$\mu_{xj} \frac{\partial}{\partial x}\left(\frac{1}{\mu_{zj}} \frac{\partial}{\partial x} E_{yj} \right) + \left(\omega^2 \varepsilon_0 \varepsilon_{yj} \mu_0 \mu_{xj} \right) E_{yj} = \beta^2 E_{yj}, \quad (1)$$

$$H_{xj} = -\frac{\beta}{\omega \mu_0 \mu_{xj}} E_{yj}, \quad H_{zj} = \frac{i}{\omega \mu_0 \mu_{zj}} \frac{\partial}{\partial x} E_{yj}. \quad (2)$$

For TM mods $\vec{H} = (0, H_{yj}(x,z), 0)$:

$$\varepsilon_{xj} \frac{\partial}{\partial x}\left(\frac{1}{\varepsilon_{zj}} \frac{\partial H_{yj}}{\partial x} \right) + \left(\omega^2 \varepsilon_0 \varepsilon_{xj} \mu_0 \mu_{yj} \right) H_{yj} = \beta^2 H_{yj}, \quad (3)$$

$$E_{xj} = \frac{\beta}{\omega \varepsilon_0 \varepsilon_{xj}} H_{yj}, \quad E_{zj} = -\frac{i}{\omega \varepsilon_0 \varepsilon_{zj}} \frac{\partial}{\partial x} H_{yj}. \quad (4)$$

Here ω is the frequency of radiation, β is the search eigenvalue, $\vec{E}$ and $\vec{H}$ are eigenvector functions, representing configurations of the electric and magnetic fields, corresponding to the eigenvalue β, ε_0 is a permittivity and μ_0 is a permeability of free space. More detailed information about the 2D model and the solve of equations (1) - (4) can be found in (Chengkun Chen et al., 2000).

With full 3D anisotropy for each "j-th" layer, the media of which is described with a non-diagonal permittivity tensor, the EM wave propagation is described by Maxwell's complete system of equations. According to the results in paper (Ivanov I., 2015) it was proved that clear TE and TM modes no exist.

This paper studies and compares the applicability and effectiveness of the most commonly used methods for calculating modes of 2D and 3D multilayer waveguides. These methods are: TMM – transformation matrix method, FD – finite difference method and Galerkin's method.

2. NUMERICAL METHODS

2.1 Transformation Matrices Method

It is assumed that in each "j-th" layer the solution is a smooth function of the type:

$$\varphi(x) = C_j \exp\left[(k_0^2 n_j^2 - \beta^2)(x - x_j)\right], \qquad (5)$$

where $n_j^2 = \varepsilon_j(x, y)$ is the refractive index, d_j is the thickness of "j-th" layer, $x_j \leq x \leq x_{j+1}$. Let $\tau_j = 1$ for TE, $\tau_j = n_j^{-2}$ for TM modes. Let $k_j = \left(k_0^2 n_j^2 - \beta^2\right)$. Then for the unknown function $\varphi(x_j) = \varphi_j$ and its derivative, $\dfrac{d\varphi_j}{dx}$ we have:

$$\begin{bmatrix} \varphi_{j+1} \\ \tau_{j+1}^{-1} \dfrac{d\varphi_{j+1}}{dx} \end{bmatrix} = \mathbf{T}_j \begin{bmatrix} \varphi_j \\ \tau_j^{-1} \dfrac{d\varphi_j}{dx} \end{bmatrix}, \qquad (6)$$

where transformation matrix $\mathbf{T}_j$ has the form:

$$\mathbf{T}_j = \begin{bmatrix} \cos(k_j d_j) & \dfrac{\tau_j}{k_j} \sin(k_j d_j) \\ -\dfrac{k_j}{\tau_j} \sin(k_j d_j) & \cos(k_j d_j) \end{bmatrix}. \qquad (7)$$

The total matrix of the transformation is obtained as a "n" multiplication of the matrices for each layer.

$$\mathbf{T} = \mathbf{T}_{n-1} \mathbf{T}_{n-2} \cdots \mathbf{T}_2 \mathbf{T}_1 = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix}. \qquad (8)$$

For the whole waveguide structure we have:

$$\begin{bmatrix} \varphi_c \\ \tau_c^{-1} \dfrac{d\varphi_c}{dx} \end{bmatrix} = \mathbf{T} \begin{bmatrix} \varphi_s \\ \tau_s^{-1} \dfrac{d\varphi_s}{dx} \end{bmatrix}. \qquad (9)$$

The dispersion equation is the transcendent equation of the type:

$$\tau_c^{-1} i k_c t_{11} - \tau_s^{-1} \tau_c^{-1} k_s k_c t_{12} + t_{21} + \tau_s^{-1} i k_s t_{22} = 0. \qquad (10)$$

2.2 Finite Difference Method

The FD method is a method approximating the value of the desired function in a given node by a linear (nonlinear) combination of values in selected surrounding nodes. Its main idea is based on the representation of derivatives by finite differences, using different differential schemes. The more nodes enter the differential template, the better the precision of convergence. In this way, the solution of the differential equations system is replaced by solving an algebraic system. Another important advantage is that the matrix derived from the coefficients of the algebraic system has a band structure.

2.3 Galerkin's Method

The numerical method of Galerkin is used with a finite number of dimensions Hilbert space $\mathbf{H}_n$ and the approximate solution of equation $\hat{A}u = f$ (operator $\hat{A}$ is normalized and limited) is searched by a finite number of orthogonal functions $\{\varphi_i\}_{i=1}^n$ as:

$$u_n = \sum_{k=1}^n a_k \varphi_k \qquad (11)$$

A linear algebraic system is obtained to determine the coefficients a_k.

$$\sum_{i=1}^n a_i \left(\hat{A}\varphi_i, \varphi_k\right) = \left(f, \varphi_k\right), \ k = 1, 2, \cdots, n. \qquad (12)$$

To determine the spectrum $f = \lambda u$, the homogeneous system needs to be solved:

$$\sum_{i=1}^{n} a_i \left[\left(\hat{A}\varphi_i, \varphi_k \right) - \lambda \delta_{ik} \right] = 0, k = 1, 2, \cdots, n \quad (13)$$

where $\delta_{ik} = \begin{cases} 0, & i \neq k \\ 1, & i = k \end{cases}$.

For a solution it is necessary that

$$D = \begin{vmatrix} (\hat{A}\varphi_1, \varphi_1) - \lambda & (\hat{A}\varphi_2, \varphi_1) & \cdots & (\hat{A}\varphi_n, \varphi_1) \\ (\hat{A}\varphi_1, \varphi_2) & (\hat{A}\varphi_2, \varphi_2) - \lambda & \cdots & (\hat{A}\varphi_n, \varphi_2) \\ \cdots & \cdots & \cdots & \cdots \\ (\hat{A}\varphi_1, \varphi_n) & (\hat{A}\varphi_2, \varphi_n) & \cdots & (\hat{A}\varphi_n, \varphi_n) - \lambda \end{vmatrix} = 0 (14)$$

It is usually difficult to select classical basic functions to meet boundary conditions and complex geometry at the boundaries of the areas under consideration, and to provide consistent convergence across the research area. This necessitates the use of pseudo-orthogonal functions with a local carrier.

The matrix of the algebraic system often has a sparse and band structure.

3. NUMERICAL EXAMPLES AND RESULTS

The numerical experiments were realized on a computer system with the following configuration: PC Lenovo, Intel Core2 Duo CPU T8400 2.26 GHz, 8 GB DDR3 RAM, HDD -160 GB / 5400.

For TMM Eigenvalues and Eigenvectors are searched using the Muller's method. (Newton's iteration method with 3-point interpolation). After FD and Galerkin discretization eigenvalues are searched using the dynamic Shifted Inverse Power method.

The comparison of the methods is made according to the following parameters:

a) Applicability of the method. This includes the possibilities of finding propagation constants and their respective EM wave configurations. Particular attention will be paid to the calculation of closely related eigenvalues.

b) Efficiency of the method. This includes the complexity of the algorithm, the approximate number of operations, the calculation time, the number of iterations needed, the number of variables needed, and so on.

3.1 Calculating 2D Waveguides

The following types of multilayer waveguides were computed:

a) Dielectric layers only. Results are shown in Table 1.

b) A combination of metal, dielectric and semiconductor layers. Results are shown in Table 2.

c) ARROW waveguides and other multi-layered structures with closest to value distribution constants. Results are shown in Table 3 and in Table 4.

3.2 Calculating 3D Waveguides

Here we apply the Finite Difference Method (FD) and the Galerkin method for calculating modes of nematic liquid crystal (LC) 5CB anisotropic channel waveguide, proposed and calculated using a pseudospectral method in (Chia-Chien Huang, 2010). In this case, the application of the TMM method is impossible. Results are shown in Table 5.

TABLE 1. Comparison of methods for calculate TE modes for waveguide structure with 6 dielectric layers proposed in (Rzhanov et al., 2010).

Method	Time to Calculate All Eigenvalues and All Eigenvectors	Minimum Number of Divisions of Computer Area for Accuracy 0.00001
Transformation Matrix Method	3.3561	6
Finite Difference Method	1.1862	512
Galerkin's Method	1.8311	512

TABLE 2. Comparison of methods for calculate modes for waveguide structure with dielectric, metal and semiconductor layers proposed in (Offersgaard J. F., 1995).

Method	Time to Calculate All Eigenvalues and All Eigenvectors	Minimum Number of Divisions of Computer Area for Accuracy 0.00001
Transformation Matrix Method	4.1632	6
Finite Difference Method	5.8311	4096
Galerkin's Method	9.1632	4096

TABLE 3. Comparison of methods for calculate modes for 9 layers ARROW waveguide structure proposed in (Chengkun Chen et al., 2000).

Method	Time to Calculate All Eigenvalues and All Eigenvectors	Minimum Number of Divisions of Computer Area for Accuracy 0.00001
Transformation Matrix Method	218.233	11
Finite Difference Method	121.334	4096
Galerkin's Method	123.093	4096

TABLE 4. Comparison of methods for calculate modes for 19 layers $Ga_xAl_{1-x}As$-AlAs waveguide structure that generates 9 very close TE-effective indexes, proposed in (Chengkun Chen et al., 2000).

Method	Time to Calculate All Eigenvalues and All Eigenvectors	Minimum Number of Divisions of Computer Area for Accuracy 0.00001
Transformation Matrix Method	96.5671	21
Finite Difference Method	59.7581	4096
Galerkin's Method	61.3833	4096

TABLE 5. Comparison of numerical methods for calculate the first 7 modes at twist angle (0^0, 30^0, 45^0, 60^0, 90^0) for nematic LC (5CB) anisotropic channel waveguide that generates very close effective indexes, proposed in (Chia-Chien Huang, 2010).

Method	Time to Calculate All Eigenvalues and All Eigenvectors	Minimum Number of Divisions of Computer Area for Accuracy 0.01
Transformation Matrix Method		
Finite Difference Method	1768.35	320 by 320
Galerkin's Method	1832.11	320 by 320

4. CONCLUSIONS

The numerical experiments we have done lead to the following conclusions:

1) The Galerkin and FD methods are applicable for calculating modes of all types of 2D and 3D waveguides.
2) The TMM method is applicable only to 2D structures or to a special type of 3D structures with a diagonal tensor of pemittivity.
3) For 2D waveguides with dielectric layers, the FD method was most effective, followed by the Galerkin method.
4) For 2D waveguides with a combination of a few number of metal, semiconductor

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and dielectric layers, the most efficient method is TMM.

5) For ARROW waveguides and 2D structures with a large number of layers, the FD and Galerkin methods are almost equivalent in efficiency, the application of TMM leads to loss of precision and the appearance of redundant roots.

6) For a 3D waveguide channel with full anisotropy, the FD and Galerkin methods show close results, assuming that Galerkin's method is more resilient when changing the number of divisions of the computing area.

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