Analysis of Time Series Geophysical Data through the Least Squares Method. Study of Spectral Characteristics

Rumiana Bojilova

National Institute of Geophysics, Geodesy and Geography
Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Bl. 3, Sofia, Bulgaria

Abstract. A significant part of the geophysical processes are represented by time series measurements of a given geophysical magnitude, performed at different times. In many cases, it is necessary to look for periodic processes for which their period is not known in advance. Due to the fact that the condition of the times when the measurements are performed are not always fulfilled, the use of the known methods of spectral analysis is not possible. In such cases, it is appropriate to use the Lomb method, which allows spectrum of the magnitude considered in the case of non-labeled data having gaps. This method is based on an approximation based on the smallest square criterion. Examples of using this method for different geophysical data are provided in this paper.

Keywords: Geophysical data, Spectral characteristics, Lomb spectrum.

1. INTRODUCTION

The spectral analysis is a method that allows to determine the presence of periodic dependencies in the studied natural processes, represented by some row of measured values of a given physical magnitude/ value. In mathematics specters are usually identified with the Fourier transform, which transforms a function of time into another function depending on period or frequency. In very specific cases, the Fourier transform can be applied to a specific timeline of data and to obtain the spectrum of the magnitude studied. But there are cases where the use of this method proves impossible - these are the cases in which the measured values are at an uneven moment. A similar problem occurs when measurements are made at the same time intervals, but not all measurements give valid values. In this case, the use of Fourier methods requires data to be interpolated, but this is not always possible with a sufficient degree of reliability.

The proposed Lomb method in 1975 allows these difficulties to be avoided. The method is based on the approximation of the order of data with sinusoidal functions, with a range of predefined ranges, the period as a criterion for the best approximation is the minimization of the sum of the squares of the differences. Approximation methods under this criterion are referred to briefly as the smallest squares method, which term will be used further in this study.

2. THEORETICAL BASIS AND METHODOLOGY

The Lomb spectrums are based on the presentation of a time series of data of the type: $y_0,y_1,y_2,...,y_{n-1}$ at moments in time respectively $t_0,t_1,t_2,...,t_{n-1}$. These moments in time are randomly located/ situated – i.e. there is no requirement for them to be equidistant.

The goal is to present the data of the type:

$$y_i(t_i) \approx Y_0 + Y_1 \cos\left(\frac{2\pi}{T} t_i\right) + Y_2 \sin\left(\frac{2\pi}{T} t_i\right)$$

The period of sine wave is set. The coefficients Y_0, Y_1, Y_2 must be determined by fulfilling the smallest square criterion, namely the sum of the squares of the differences between the approximation and the measured values being minimal.

$$S = \sum_{i=0}^{n-1} \left[Y_0 + Y_1 \cos\left(\frac{2\pi}{T} t_i \right) + Y_2 \sin\left(\frac{2\pi}{T} t_i \right) - y_i(t_i) \right]^2$$

To solve this task a necessary and sufficient condition is to be fulfilled (Korn and Korn, 2000.):

$$\frac{\partial S}{\partial Y_0} = 0; \frac{\partial S}{\partial Y_1} = 0; \frac{\partial S}{\partial Y_2} = 0;$$

The solution to the problem is to solve a system of linear equations (Luenberger, D. G., 1969):

$$\left| \begin{array}{l} nY_0 + Y_1 \sum_{i=0}^{n-1} \cos\left(\frac{2\pi}{T} t_i \right) + Y_2 \sum_{i=0}^{n-1} \sin\left(\frac{2\pi}{T} t_i \right) = \sum_{i=0}^{n-1} y_i(t_i) \\ Y_0 \sum_{i=0}^{n-1} \cos\left(\frac{2\pi}{T} t_i \right) + Y_1 \sum_{i=0}^{n-1} \cos^2\left(\frac{2\pi}{T} t_i \right) + Y_2 \sum_{i=0}^{n-1} \sin\left(\frac{2\pi}{T} t_i \right) \cos\left(\frac{2\pi}{T} t_i \right) = \sum_{i=0}^{n-1} y_i(t_i) \cos\left(\frac{2\pi}{T} t_i \right) \\ Y_0 \sum_{i=0}^{n-1} \sin\left(\frac{2\pi}{T} t_i \right) + Y_1 \sum_{i=0}^{n-1} \cos\left(\frac{2\pi}{T} t_i \right) \sin\left(\frac{2\pi}{T} t_i \right) + Y_2 \sum_{i=0}^{n-1} \sin^2\left(\frac{2\pi}{T} t_i \right) = \sum_{i=0}^{n-1} y_i(t_i) \sin\left(\frac{2\pi}{T} t_i \right) \end{array} \right.$$

In order to obtain a spectrum of the given time-line data, the described procedure is performed for an interval from T1 to T2. The amplitudes of approximations for each T period are:

$$A(T) = \sqrt{Y_1^2(T) + Y_2^2(T)}$$

To justify the significance of the detected spectral components, whose period is the period of actually occurring periodic processes in the physical magnitude considered, it is appropriate for each spectral component to calculate the signal / signal + noise (Djermanova & Djermanov, 2009) in a quadratic sense, which corresponds to power at power quantities. In this case, the dispersion of the approximating sine wave with the corresponding period is calculated and subdivided into the dispersion of the entire data row.

3. EXPERIMENTAL RESULTS

An analysis of periodic variations in geomagnetic activity can serve as an example of spectral analysis based on Lomb spectrum. The period from January to April 2008 is considered. Under conditions of low solar and geomagnetic activity, in some cases there is a long position of the coronal "holes" of the sun at about 120°. In such cases, due to the Sun`s rotation period being about 27 days long the plasma streams emitted by these coronal holes cause 9-day variations in the Earth's geomagnetic activity (Mukhtarov and Pancheva, 2012).

Fig. 1 shows the spectrum of the planetary index of geomagnetic activity Kp, in which the presence of a 9-days (216 hour) spectral component in which the signal / (signal + noise) is maximal is shown. The main spectral component is also present in the spectrum - 27 days and its second harmonic - about 13.5 days, but with significantly smaller amplitudes than the 9-day period. This is due to the fact that the coronal holes are not located exactly at 120° and their intensity is not exactly the same is different from what it is in other seasons.

The variations of the geomagnetic activity represented by the planetary index Kp are also shown in the magnetic fields of specific geomagnetic observatories. Represented to Fig. 2 spectrums of the three Cartesian components of the Earth's magnetic field contain the same spectral components as the planetary index. They have the highest amplitude in the X-component (parallel to the geographical meridian), which practically coincides with the horizontal component which is most influenced by the solar wind for the coordinates of Panagurishte station (42° 30,9′ N, 24° 10,6′ E).

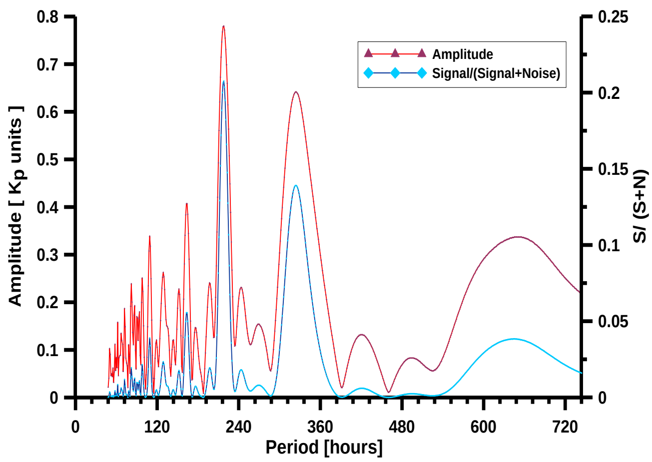

Fig. 1. Spectrum of the planetary index Kp

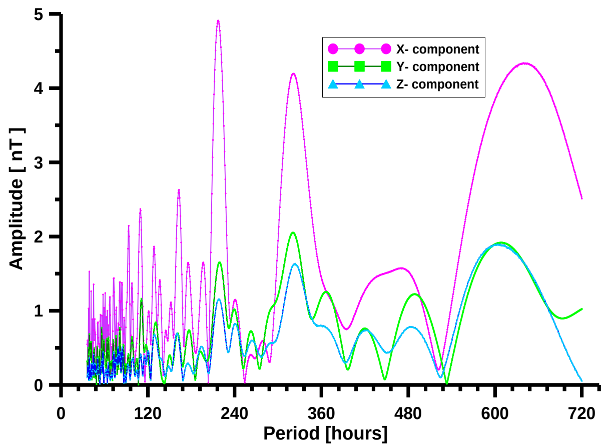

Fig. 2. Spectrums of the cartesian components

Variations in geomagnetic activity, even if they do not reach the level of a geomagnetic storm, affect the electron concentration of the ionosphere. Fig. 3 (upper plot) shows a part of the 24-hour movement of the critical frequencies of the ionospheric area F (*foF2*), measured by the Juliusruh / Ruegen station (54 ° 37'N, 13 ° 22'E). Fig. 3 (bottom plot) shows the spectrum of critical frequencies at the Juliusruh station for the reference period.

A time interval is displayed for which there are gaps in the data as reflected in the holes in Fig. 3 (upper plot). In the spectrum illustrated in Fig. 3 (bottom plot) a significant spectral component with a period of 9 days is clearly observed.

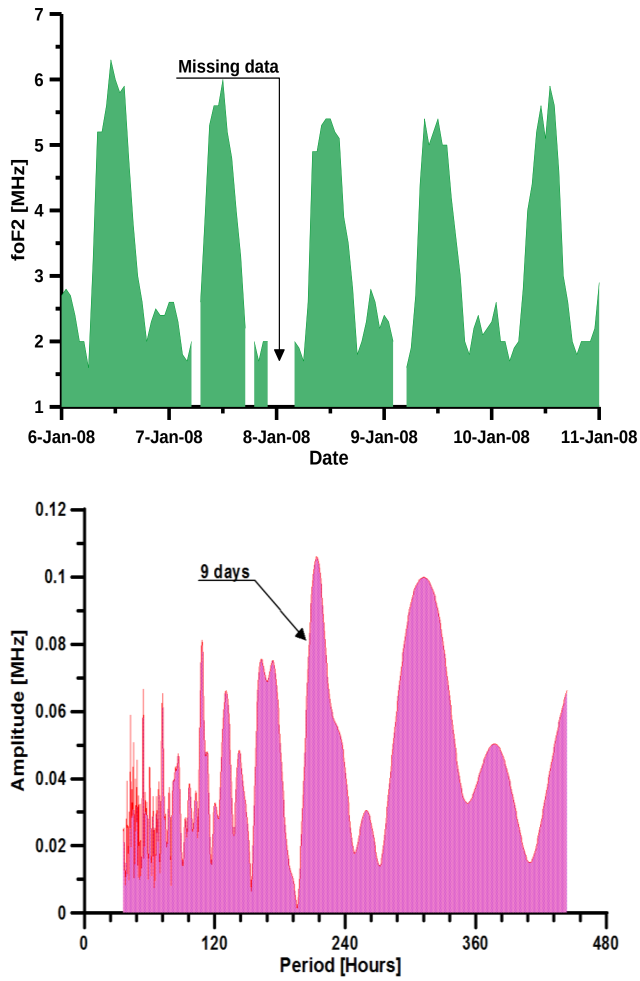

Fig. 3. Movement of the foF2 critical frequencies measured at the Juliusruh / Rugen station (upper plot) and their spectrum (bottom plot).

4. CONCLUSIONS

The use of Lomb spectrums based on the smallest squares method allows for spectral analysis of time series data. The examples presented illustrate the possibility to explore their relationship through a comparison of the spectrums of different geophysical quantities.

Such Lomb-based operations are widespread and of paramount importance for any type of data that contains invalid values and time-lag values.

REFERENCES

Djermanova N., Djermanov I., "Fundamentals of Electronics. Instruments, circuits and tasks with comments and directions for laboratory practice", (Основи на електрониката. Прибори, схеми и задачи с коментари и упътвания за лабораторен практикум), WiNi 1837, 2009

Hansen, A. J., T. Walter, and P. Enge. "Ionospheric correction using tomography." *Proceedings of Institute of Navigation ION GPS-97*, Kansas City (1997): 249-260.

Kutiev, I., P. Muкhtarov, Empirical modeling of global ionospheric foF2 response to geomagnetic activity, *JOURNAL OF GEOPHYSICAL RESEARCH*, VOL. 108, NO. A1, 1021, doi: 10.1029/2001JA009134, 2003

Mukhtarov, P., N. Penov, D. Pancheva, N(h) PROFILES DERIVED FROM IONOGRAMS AND THEIR APPLICATION FOR STUDYING MID-LATITUDE IONOSPHERIC RESPONSE TO GEOMAGNETIC STORMS, *Comptes rendus de l'Acad´emie bulgare des Sciences*, Tome 66, No9, 2013

NOAA Space Weather Scales, http://www.swpc.noaa.gov/noaa-scales-explanation

Pancheva, D., P. Mukhtarov, B. Andonov, Global structure of ionospheric TEC anomalies driven by geomagnetic storms, *Journal of Atmospheric and Solar-Terrestrial Physics* 145 (2016) 170–182, www.elsevier.com/locate/jastp