Multifaceted Connections Between Physics and Mathematics

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Abstract. Just as the phenomena in the material world are in complex and multifaceted relationships with each other, so the sciences that study these phenomena are completely linked. Physics has the closest connection with mathematics. Mathematical concepts and methods are applied in every research in physics, and the discovery of new phenomena in physics has become an occasion for the development of whole new branches in mathematics itself. This report examines the need for timely training of a certain mathematical apparatus as a prerequisite for the assimilation and understanding of any physical theory in the middle stage of school education. Examples of interesting connections between physics and mathematics will be presented.

Keywords: phenomena, physics, mathematics

1. INTRODUCTION AND AIMS

Society today is ever more dependent upon technological competence for the sustainable development of health, industry and environment. Consequently society needs technologically competent citizens (Osborne & Dillon, 2008). At the heart of such technological competence is physics, and physics necessitates mathematical competence (Redish, 2006). Physics and mathematics are strongly interrelated, and this relationship becomes more apparent as students advance in school. Mathematics is more than just a tool for working with physics problems; the discourse of physics is mathematical in nature (Uhden et al., 2011).

1. 1. Mathematics is a Powerful Tool for Summarizing Physical Concepts and Laws

This has been known since ancient times. Thanks to mathematical relationships in a right triangle, Eratosthenes determined the radius of the Earth. Eratosthenes supposed that: a) The Earth is round in shape. b) Alexandria and Aswan lie on the same meridian (that is, they are directly under each other as viewed from the pole).

Under these assumptions, both cities are points on a circle. The arc between them (l) is the distance between the cities. The angle (θ) was measured using the shadow of the lighthouse at Alexandria on a day when the Aswan wells cast no shadow. Then, using his mathematical knowledge, he evaluated that $\frac{H}{d} \approx \frac{l}{R}$ and $R = 46250$ km. The obtained value is very close to the one received by the most modern methods.

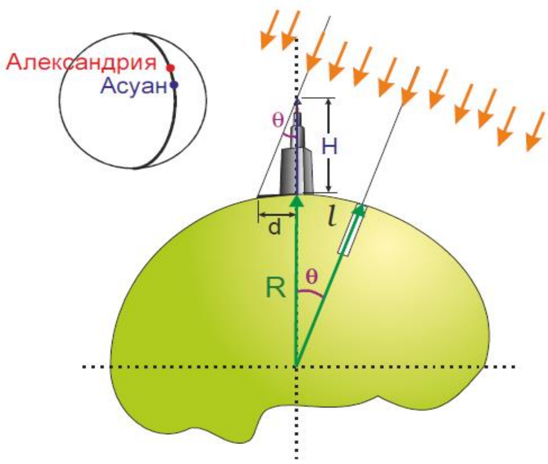

Fig. 1 Images of Earth, Alexandria and Aswan.

1. 2. Studying Physics in Secondary School

In school, the study of mathematics begins earlier than the science of physics, and thus it provides the means and methods for the general and accurate expression of the relationships between physical quantities revealed as a result of experiment or theoretical research. For the correct use of the mathematical apparatus, it is necessary for the physics teacher to know in detail the school mathematics course, the accepted terminology and interpretation of the material, in order to work in a “mathematical language” familiar to the students. The connections between physics and mathematics are so many that we can hardly cover all the possibilities for their dismantling. However, the late learning of some concepts (in the second stage of secondary education) or the lack of coherence in the physics and mathematics programs in the school course pose limitations. In the implementation of lessons with cross-curricular connections, these limitations can be overcome by giving preference not so much to strict mathematical proofs, but to their clarity. Concepts such as function, quantity, derivative, integral, vector, etc. can be used. This report examines examples of some important mathematical concepts and knowledge necessary for the assimilation and understanding of given physical quantities in the middle stage of school education.

EXAMPLE: VECTORS

The concept of a vector is introduced in the 8th grade within several lesson units. In the same class, physical quantities represented by vectors are also studied. Examples are: speed, acceleration, force, etc.

Thus, the lack of fine-tuning between the physics and mathematics programs results in losses for both sciences. Mathematics is understood by students only through concrete examples of its application in life, which can be realized precisely through physical phenomena and laws. A deep understanding of physical phenomena depends on knowledge of mathematics. An important methodological moment is the clarification of the specifics of the general definitions of the concepts in mathematics when they are applied in physics.

For example, a vector retains its general meaning defined in mathematics, but it is important to say, for example, that physics also introduces the need for an application point (otherwise the ball will not be hit) i. e. not every directed stroke, although of the same direction and magnitude (same force) would cause the ball to move.

Fig. 2 Picture demonstrating an application point.

EXAMPLE: OPERATIONS WITH VECTORS

The SUMMATION OF VECTORS and THEIR SCALAR AND VECTOR PRODUCT are also widely used in physics.

What more elegant way to explain the difference and application of the product of vectors than their meaning in finding work and in rotational motion? Work is represented by the scalar product of force and displacement:

$$A = \vec{F}.\vec{l} \tag{1}$$

The result is a scalar (number). An interesting point that arises when discussing this work is the need for a COS function for an analytical solution to the problem: $A = |\vec{F}||\vec{l}|\cos\theta$.

Rotary motion demonstrates the application of the vector product:

$$\vec{v} = \vec{\Omega} \times \vec{r} \tag{2}$$

where

$\vec{v}$ is the linear velocity,

$\vec{\Omega}$ is the angular velocity,

$\vec{r}$ is the radius of gyration.

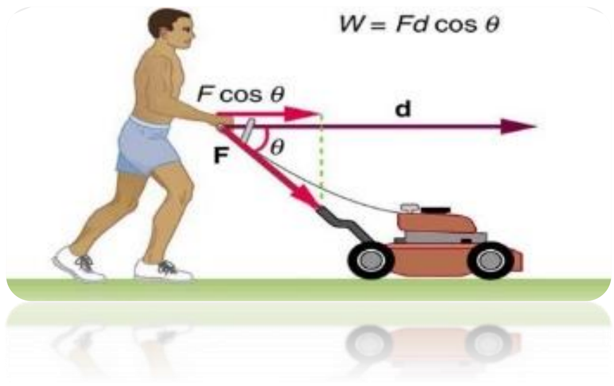

Fig. 3 Analytical solution.

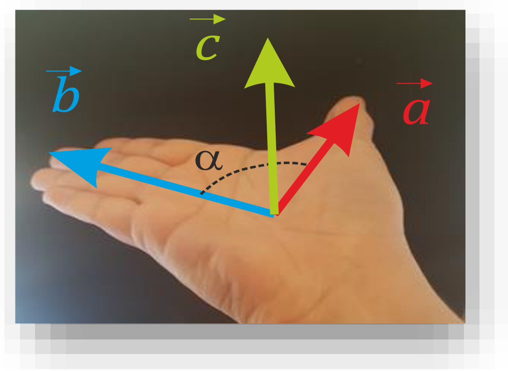

Fig. 4 Rotary motion.

Analytically, the magnitude of the vector $\vec{v}$ is obtained by using the SIN function

$$\vec{c} = |\vec{a}||\vec{b}|\sin\alpha \tag{3}$$

EXAMPLE: TRIGONOMETRICAL FUNCTIONS

As it turns out, even the product of vectors is enough to show the need to understand trigonometric functions.

However, they find extremely wide application in physics and are very suitable to be studied before, rather than after, the wave material, for example.

Examples of the need for such knowledge are: oscillation of a pendulum, waves – mechanical, electro-magnetic and many others.

Electromagnetic waves are part of our everyday life and are an extremely appropriate example of the application of the trigonometric functions sine and cosine.

The figure shows an EM wave with its electric and magnetic parts.

In addition to the mathematical knowledge, the characteristics such as amplitude, wavelength, frequency, etc., which are related to the application of these waves are added here. This is how the spectrum of EM waves is introduced, as waves of different frequencies.

2. CONCLUSION

The multifaceted connections between physics and mathematics are at the core of our knowledge of nature.

With the help of appropriate synchronization of the learning content studied in the two subjects at school, a deep and lasting understanding of the phenomena and processes in nature can be achieved.

This, in addition, is an excellent motivation for students in learning the secrets not only of nature, but also of mathematics.

REFERENCES

Redish, E. F. (2006). Problem solving and the use of math in physics courses. Arxiv: 0608268.

Uhden, O., Karam, R., Pietrocola, M., and Pospiech, G. (2011). Modelling Mathematical Reasoning in Physics Education. Science & Education, pp. 1-22.