3D Model of Thermal Interaction of a Laser Radiation with Multilayered Biological Oobject

Emine Alieva and Ivan Ivanov

Faculty of Physics and Technology, University of Plovdiv, Tsar Asen str., 24, Plovdiv, Bulgaria

Abstract. The proposed 3D model studies the thermal effect of laser radiation on a multilayer biological object. Medium re-emission, absorption, and convection processes have been reported. To solve the model, the FDM (Finite Difference Method) and the FEM (Finite Element Method) are used for different number of divisions of the computing area. Numerical simulations have been created to study the temperature field generated by the Er:YAG laser and CO_2 laser on a tooth part consisting of enamel and dentine.

Keywords: nonlinear thermal conductivity equation, heat equation, numerical analysis.

1. INTRODUCTION

Temperature distribution is one of the most important parameters in laser technology used in medicine. The thermal properties of the biological objects depend on their multi-layered structure as well as on the respective thermal capacity and thermal conductivity on each layer. With the current mass distribution of medical lasers, it is particularly necessary to create a model (and a method of solving it) to find optimal research parameters and optimal laser processing modes for the various layers with minimal warming of the surrounding tissues.

2. PHYSICAL MODEL

Thermal process modeling of the biological structure allows optimizing the heating of surrounding tissues thus enhancing the quality of laser biotechnology. The absorbed energy of laser radiation (q_{laser}) causes a local rise in the temperature in the irradiated area. Part of the heat leaves the processing area due to thermal conductivity (q_{cond}) in the surrounding biological tissues, causing undesired heating. Another part of the heat leaves the processing area due to the convection (q_{conv}) and the effect of the re-radiation losses (q_{rad}) in the surrounding space. It should be noted that the heat dissipation should be considered together with the re-radiation losses, because even at 60 ° C they start to dominate the convection.

Attenuation of laser radiation q_{laser} due to absorption from media is:

$$q_{laser}(z) = q_0(1-R)e^{-\alpha z}, \quad (1)$$

where q_0 is the initial value of the power density of the laser beam, R is the reflection coefficient, and α is the coefficient of extinction.

By q_{rad} we present the density of the heat flow from the radiating processes:

$$q_{rad} = \varepsilon\sigma(T^4 - T_0^4), \quad (2)$$

where ε is a radiation-related factor, σ is the Stephan-Boltzmann's constant, T is the desired temperature distribution, and T_0 is the temperature of the surrounding space.

q_{conv} is the density of the heat flow due to the convection, (h convection coefficient) :

$$q_{conv} = h(T - T_0), \quad (3)$$

The temperature distribution is described by the general heat equation:

$$c\rho \frac{\partial T}{\partial t} - \nabla \cdot \left(k \nabla T \right) = q_{laser} + q_{rad} + q_{conv} , (4)$$

where c is the specific heat of the layers, ρ is the density of the medium of each layer, t is the time, k is the coefficient of thermal conductivity.

3. MATHEMATICAL MODEL

The mathematical model corresponding to the physical Eq.(1) – Eq.(4), having in mind the symmetry in the direction 0*x* and 0*y*, is given for the two-dimensional nonstationary case of anisotropic media (along the 0*z* axis) in the following way:

$$c(x,z)\rho(x,z)\frac{\partial T(x,z,t)}{\partial t} - \left(\frac{\partial}{\partial x} k\left(x,z\right) \frac{\partial}{\partial x} T\left(x,z\right) + \right.$$

$$\left. + \frac{\partial}{\partial z} k\left(x,z\right) \frac{\partial}{\partial z} T\left(x,z,t\right) \right) = q\left(x,z,t\right). \qquad (5)$$

Boundary conditions:
At the region of interaction ($z = 0$) :

$$k \frac{\partial T(x,0,t)}{\partial z} = h(T - T_0) + \varepsilon \sigma \left(T^4 - T_0^4 \right). \ (6)$$

At the bottom at $z = z_{\max}$ and at the sides at $x = 0$ and $x = x_{\max}$ we assume that:

$$T = T_{body}. \ (7)$$

Initial condition at $t = 0$:

$$T(x,z,0) = T_{body}.$$
(8)

4. NUMERICAL MODEL

To solve model Eq.(5) - Eq.(8) we created a program using an implicit differential scheme by the finite difference method (FD), where computer region division was created with a rectangular unequal net.

To solve the non-linear algebraic system obtained after the discretization we used the Newton-Raffson method.

We also solved the model Eq.(5) – Eq.(8) using the finite element method (FE) through the capabilities of the PDETOOL module, part of the MATLAB 2015a.

5. NUMERICAL EXPERIMENTS

We made numerical simulations for the thermal impact of two different laser sources (Er:YAG and CO2) on the tooth, which consists of enamel and dentin. We presented the tooth as a two-layer system consisting of a layer of ENAMEL with a thickness of about 20 µm and a layer of DENTIN with a thickness of 100 - 120 µm. The thermodynamic and spectral characteristics according to (Featherstone, et al., 2000) and (Malukov, et al., 2012) are the following:
$T_{body} = 36.8^0 C, T_0 = 20^0 C, \varepsilon = 0.95, \sigma = 5.67 \cdot 10^{-8}$
Both lasers have 2 watts of power, and a focussed Gaussian beam with a diameter of 2 mm.

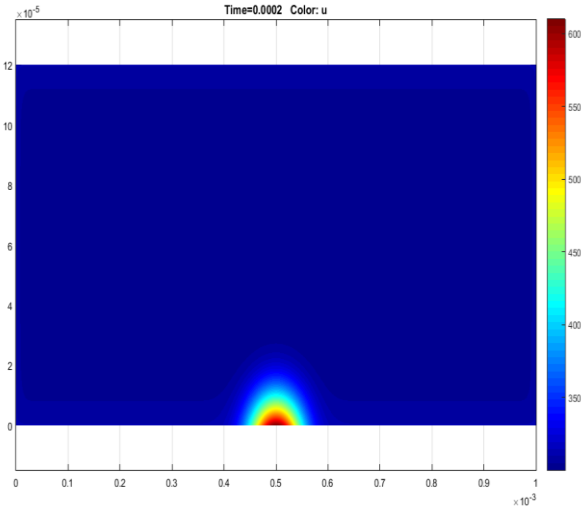
TABLE 1. Thermodynamic and spectral characteristics of tooth for Er:YAG laser treatment, $\lambda = 2.94\ \mu m$

	c $\left[\frac{J}{kgK}\right]$	k $\left[\frac{W}{mK}\right]$	ρ $\left[\frac{kg}{m^3}\right]$	R	α
Enamel	970	1.0	2950	0.05	800
Dentin	1430	1.0	2180	0.05	2480

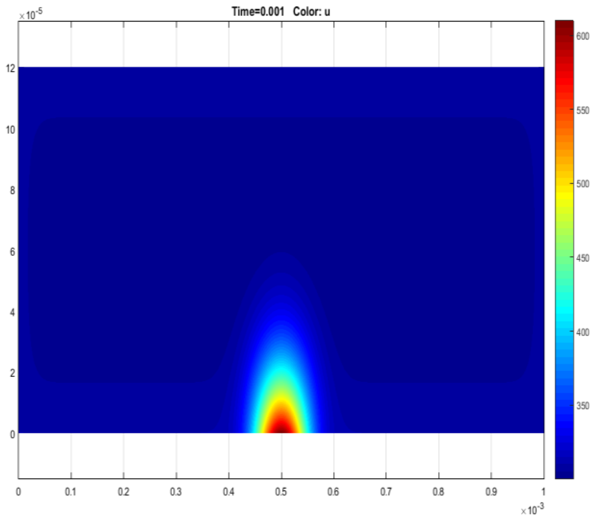
TABLE 2. Thermodynamic and spectral characteristics of tooth for CO_2 laser treatment, $\lambda = 10.6\ \mu m$

	c $\left[\frac{J}{kgK}\right]$	k $\left[\frac{W}{mK}\right]$	ρ $\left[\frac{kg}{m^3}\right]$	R	α
Enamel	970	1.0	2950	0.49	820
Dentin	1430	1.0	2180	0.49	540

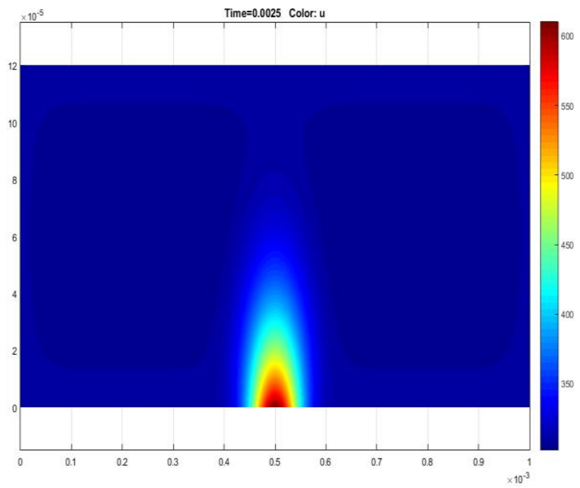
In the heat dynamics study, as shown in Figure 1, it was found that after some time the process became quasi-stationary for both laser treatments.



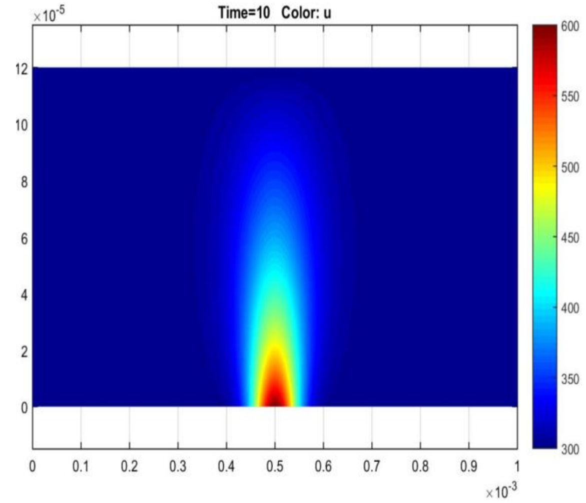
a) t = 0.0002 s



b) t =0.0010 s



c) t = 0.0250 s

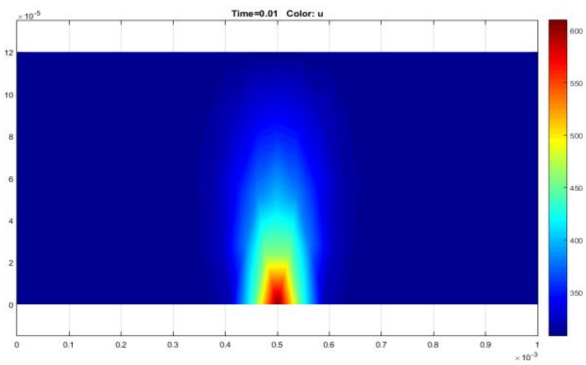


d) t = 0.0100 s

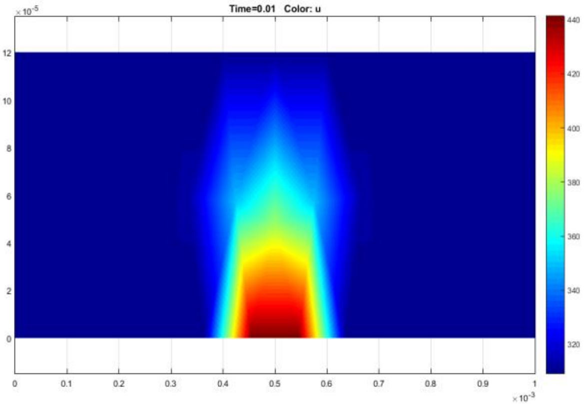
Fig. 1 Dynamics of $T(x,z,t)$ at time.

The determination of the accuracy and resilience of the solution is made by comparing the calculated temperature field profiles $T\left(x,z,t\right)$ with different number of divisions (10, 20, 40, 80, 160 and 320) by coordinates x and z. It is established, as shown in Figure 2, by repeating the type and size of the shape and the values (color) in the net with the sufficient number of divisions (usually greater than 80).

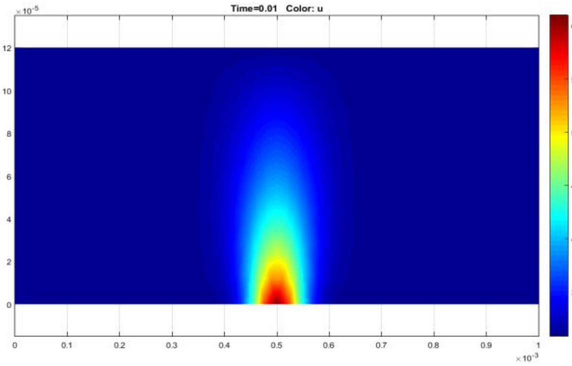
After the time of 0.01 seconds, when the process become practically stationary, we compared the depth of penetration with the CO2 laser and the Er:YAG laser.



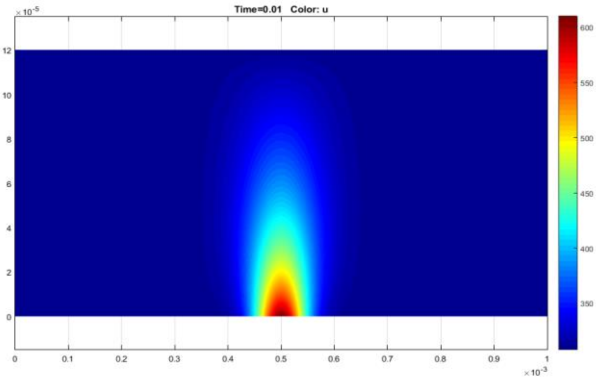
a) 10 divisions



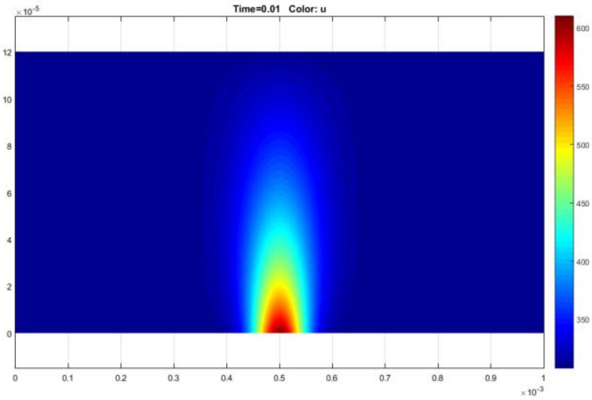
b) 20 divisions



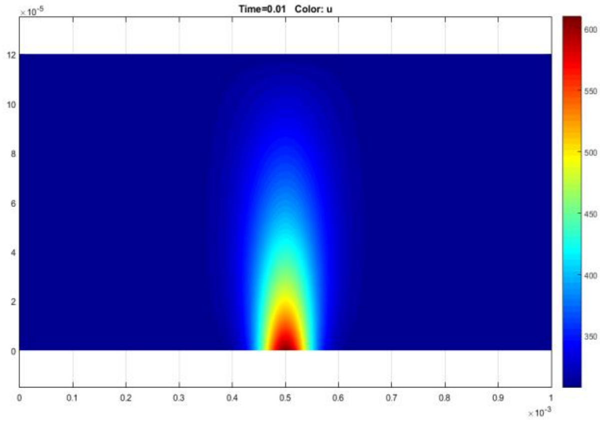
c) 40 divisions



d) 80 divisions



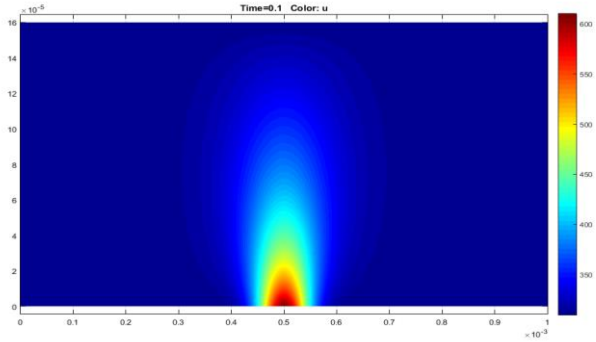
e) 160 divisions



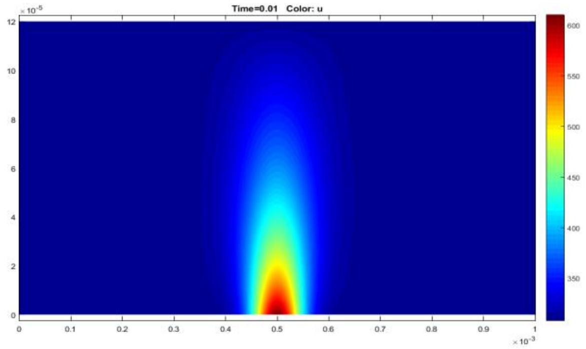
f) 320 divisions

Fig. 2 Profile of $T(x,z,t)$ with different number of divisions of the computed area by x and z. Time step is 0.0001 s.

We have found, as shown in Figure 3, that zones with the same temperature are deeper when treated with CO_2 laser than with Er:YAG laser treatment.



a) CO_2 laser



b) Er:YAG laser

Fig. 3 Comparison of penetration depth with laser treatment.

6. COMMENTS AND CONCLUSIONS

A common physical model of the thermal impact of an EM wave with a multilayered biological object was proposed, taking into account the influences of the heat sources associated with the re-emission from the media and the convection. We have implemented persistent numerical models based on the finite difference method (implicit scheme) and the finite elements that have successfully approximated the differential equations of the physical model. We found deeper CO_2 laser penetration at the same time of processing (after 0.01 seconds the process becoming practically stationary), which has also been proven experimentally by other authors.

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